INDIAN STATISTICAL INSTITUTE

CHENNAI CENTRE

M.STAT First Year

2016-17 Semester II

Nonparametric Inference and Sequential Analysis:
Midterm Examination

Maximum Marks 20.

Duration: 2 hours

1. Let $(X_1, ..., X_n)$ be a random sample from the uniform distribution on the interval $[0, 1]$ and let $R = X_{(n)} - X_{(1)}$, where $X_{(i)}, i = 1, 2, ..., n$ is the ith order statistic. Find the limiting distribution of $2n(1 - R)$. 5

2. Let X be positive random variable with continues cdf F . Find an estimator for the quantity defined by $\theta = \int_0^\infty (1 - F(x))^n dx$ using U-statistics theory. Obtain the asymptotic distribution of the estimator so obtained. What is the value of the asymptotic variance when $F(x) = 1 - e^{-x}$ and $n = 3$. 8

3. Define consistency of a test. If F is a continuous distribution function with unique median θ, then show that Wlicoxon signed rank test is consistent for tests on θ. 4

4. For testing the hypothesis $H_0 : F(t) = G(t)$ for all $t \in R$, show that the tests based on Wilcoxon rank sum and Mann-Whitney U-statistics are same. Find the asymptotic critical region against the test $H_1 : F(t) \neq G(t)$ for some $t \in R$ based on Mann-Whitney U-statistics. 6